## Review for MA 26500 Final Exam

**Review all of your exams and quizzes.**

**Row operations.** What are the three elementary row operations?
Know the definition of echelon form and reduced echelon form.
Know how to reduce a matrix to echelon and reduced echelon forms using elementary row operations.

**Systems.** Know how to solve a system of linear equations (using row operations on an augmented matrix) and how to write the solution of the system in parametric vector form. Know how to describe the solution geometrically.

Know how to describe the solutions of a system if you know the number of pivot columns in, or the rank of, the coefficient matrix.

Know how to determine the uniqueness and the consistency of a system in the presence of a parameter.

Know how to write a system of linear equations as a matrix equation, $A\vec{x} = \vec{b}$, and as a vector equation.

Know how to write a vector equation as a system of linear equations and as a matrix equation.

Know how to write a matrix equation as a system of linear equations and as a vector equation.

**Linear combination.** If $\vec{v}_1, \vec{v}_2, ..., \vec{v}_p$ are vectors in $R^n$, what does it mean that $\vec{b}$ is a linear combination of $\vec{v}_1, \vec{v}_2, ..., \vec{v}_p$? How do you show this algebraically? How do you represent it graphically in $R^2$?

**Span.** If $\vec{v}_1, \vec{v}_2, ..., \vec{v}_p$ are vectors in $R^n$, what does Span$\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_p\}$ mean?

How do you show that a vector $\vec{v}$ is in the Span$\{\vec{v}_1, \vec{v}_2, ...\vec{v}_p\}$?

Be able to give a geometric description for the span of one, two, or three vectors in $R^2$ or $R^3$.

**Multiplication of a matrix by a vector.** Remember that if $A = [\vec{a}_1 \ \vec{a}_2 \ ... \ \vec{a}_n]$ and $\vec{x} = \begin{bmatrix} x_1 \\ : \\ x_n \end{bmatrix}$, then the definition of matrix multiplication of a vector is $A\vec{x} = x_1\vec{a}_1 + x_2\vec{a}_2 + ... + x_n\vec{a}_n$. (So $A\vec{x}$ is a linear combination of the columns of $A$.)

Also remember that if $A = [\vec{a}_1 \ \vec{a}_2 \ ... \ \vec{a}_n]$ and $\vec{x} = \begin{bmatrix} x_1 \\ : \\ x_n \end{bmatrix}$ is a solution of the system $A\vec{x} = \vec{b}$, then the above definition of matrix multiplication of a vector tells us that $\vec{b}$ is a linear combination of the columns of $A$ and $x_1, x_2, ..., x_n$ are the weights for which we have $\vec{b} = x_1\vec{a}_1 + x_2\vec{a}_2 + ... + x_n\vec{a}_n$. It also tells us that $\vec{b}$ is in the Span$\{\vec{a}_1 \ \vec{a}_2 \ ... \ \vec{a}_n\}$, which is the same as saying that $\vec{b}$ is in Col$(A)$, which is the same as saying that $\vec{b}$ is in the range of the transformation defined by the matrix $A$.

**Algebra of matrices.** Know the algebra rules for sum, product, scalar multiplication, inverse, transpose, and determinant of matrices.

**Inverse of a $n \times n$ matrix.** Know how to find $A^{-1}$. (Recall that you row reduce $[A|I]$ to get $[I|A^{-1}]$.)

Know how to use $A^{-1}$ to solve a system $A\vec{x} = \vec{b}$.

**The invertible matrix theorem.** You should be able to give a statement equivalent to "$A$ is invertible" using each of the following concepts: (i) the homogeneous system $A\vec{x} = \vec{0}$, (ii) the system $A\vec{x} = \vec{b}$, (iii) pivot positions of $A$, (iv) rref(A), (v) Nul(A), (vi) Col(A), (vii) det$(A)$.

**Linear Transformations.** Know what it means for a transformation to be a *linear transformation.* (The transformation of a sum of vectors is the sum of the transformed vectors. And the transformation of a multiple of a vector is a multiple of the transformed vector.)

Know the definition of the domain and range of a transformation.

Know how to show that a vector is in the range of a linear transformation.

Know how to show that a transformation in not linear.

Remember that any matrix $A$ can be used to define a linear transformation, that is, the transformation given by $T(x) = Ax$. (If $A$ is $m \times n$, then $T : R^n \to R^m$.)

On the other hand, remember that every linear transformation can be defined by a matrix. If $T : R^n \to R^m$ is any linear transformation, then there is a $m \times n$ matrix $A$ such that $T(x) = Ax$, and $A$ is given by $A = [T(\vec{e}_1)\ T(\vec{e}_2)\ ...\ T(\vec{e}_n)]$. The matrix $A$ is called the standard matrix for $T$.

Know how to find the standard matrix of a linear transformation, whether the transformation is given algebraically or geometrically (for example, rotations and their rotation matrices).

**Subspaces.** Know the definition of a subspace of $R^n$. (A subset of vectors from $R^n$ that is "closed under vector addition" and "closed under scalar multiplication".)

Know how to show that a subset of $R^n$ is a subspace of $R^n$, or show that it is not a subspace.

Know the definition of the three fundamental examples of subspaces of $R^n$ namely, Col($A$), Nul($A$), and the span of a set of vectors.

Know how to show that a vector is in Col($A$).

Know how to show that a vector is in Nul($A$).

**Linear independence.** What does it mean for a set of vectors to be linearly independent and how do you show that a set of vectors is linearly independent?

**Linear dependence.** What does it mean for a set of vectors to be linearly dependent and how do you show that a set of vectors is linearly dependent? How do you find a linear dependence relation between linearly dependent vectors and how do you write one vector as a linear combination of the other vectors?

**Basis and dimension of a subspace.**
What does it mean for a set of vectors to form a basis for $R^2$ or $R^3$?
How do you show that a set of vectors form a basis for $R^2$ or $R^3$?
How do you find a basis for Col($A$), or Nul($A$), or the span of a set of vectors?
Remember that Span$\{v_1, v_2, ..., v_p\}$ = Col($B$) where $B = [v_1\ v_2\ ...\ v_p]$.
How do you find the coordinates of a vector with respect to a basis?

**Determinants.** How do you compute a determinant? (Cofactor expansion.)
Know the effect of row operations on the determinant. (Problems 15 to 20 in Section 3.2.)
Know the algebraic properties of the determinant. (Problems 39 and 40 in Section 3.2.)

**Eigenvalues and Eingenvectors.** Remember the eigenvalue, eigenvector equation $A\vec{x} = \lambda\vec{x}$.
Know how to show that a vector is an eigenvector for a matrix $A$.
Know how to show that a scalar is an eigenvalue for a matrix $A$.
Know how to find all the eigenvalues (with their multiplicities) for a matrix $A$.
Know how to find a basis for the eigenspace of an eigenvector for a matrix $A$.
Know what is meant by the "multiplicity" of an eigenvalue.

**Diagonalization.** Know when a matrix diagonalizable.

If $A$ is diagonalizable, find $P$ and $D$ such that $A = PDP^{-1}$. How is $D$ defined? How is $P$ defined?

Be able to compute the power of a matrix $A$ if $A$ is diagonalizable.