1. Be able to use the table of quadratic surfaces (pages 607-608) and "completing the square" to name and sketch a given equation of the form
$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

2. For a function of two variables $f(x, y)$, a **level curve** (or **level set**) of height c is the set of (x, y) points that solve the equation $c = f(x, y)$. Another way to define a level curve of height c is to intersect the graph of $z = f(x, y)$ with the horizontal plane $z = c$ and then project the intersection curve down to the xy-plane.

3. For a function of two variables $f(x, y)$, a **contour plot** is a simultaneous graph in the xy-plane of many level curves for f. You should be able to tell from a contour plot where the function f has a local maximum or local minimum, or where it has a "saddle point." You should also be able to tell from a contour plot where the function's graph is more or less steep.

4. For a function of two variables $f(x, y)$, a **vertical slice** is the intersection of the graph of $z = f(x, y)$ with a plane perpendicular to the xy-plane. Know how to use the idea of a vertical slice to describe what we mean by the "partial derivatives" and the "directional derivatives" of $f(x, y)$ at a given point (x_0, y_0).

5. For a function of two variables $f(x, y)$, know how to compute the partial derivatives $f_x(x, y)$ and $f_y(x, y)$, and, for a given point (x_0, y_0), know the meaning of $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$.

6. For a function of two variables $f(x, y)$, know how to compute the "higher order" and "mixed" partial derivatives.

7. For a function of two variables $f(x, y)$ and a point (x_0, y_0), know why the two vectors
$$(\,1,\,0,\,\frac{\partial f}{\partial x}(x_0, y_0)\,) \quad \text{and} \quad (\,0,\,1,\,\frac{\partial f}{\partial y}(x_0, y_0)\,)$$
are both tangent vectors to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

8. The cross product of the two vectors from the previous item,
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & f_x(x_0, y_0) \\ 0 & 1 & f_y(x_0, y_0) \end{vmatrix} = (\,-\frac{\partial f}{\partial x}(x_0, y_0),\,-\frac{\partial f}{\partial y}(x_0, y_0),\,1\,)$$
is perpendicular to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$, so it is a normal vector for the plane tangent to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

9. Let $\mathbf{n} = (\,-f_x(x_0, y_0),\,-f_y(x_0, y_0),\,1\,)$, let $\mathbf{x_0} = (x_0, y_0, z_0)$ denote a given point on the graph of $z = f(x, y)$, and let $\mathbf{x} = (x, y, z)$ denote an unknown point on the plane tangent to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$. Then the vector equation $\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0$ for this tangent plane simplifies to
$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

10. The tangent plane formula is itself a function of x and y, $T_{(x_0,y_0)}(x, y)$, that we call the **tangent plane approximation function** for $f(x, y)$ at the point (x_0, y_0),

$$T_{(x_0,y_0)}(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

For arbitrary points (x, y) "near" the given point (x_0, y_0)

$$f(x, y) \approx T_{(x_0,y_0)}(x, y).$$

The tangent plane function approximates the original function.

11. Here is a picture illustrating the tangent plane approximation.

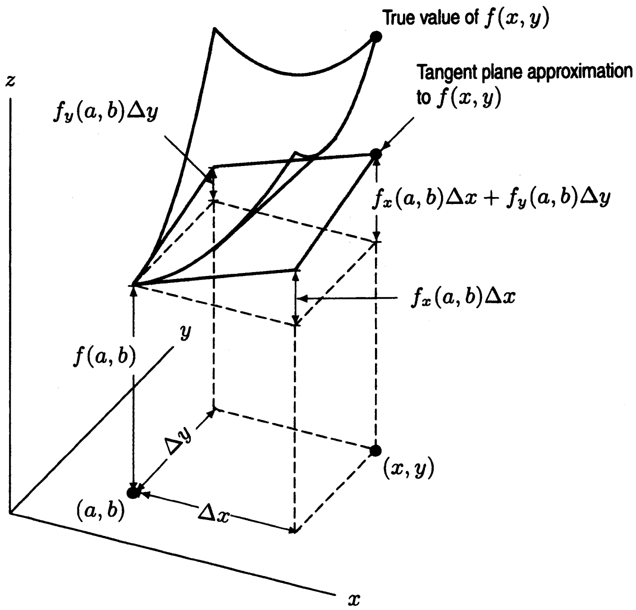

Figure 14.22: Local linearization: Approximating $f(x, y)$ by the z-value from the tangent plane

12. For a function of two variables $f(x, y)$, the **gradient vector** is defined to be the two-dimensional vector

$$\nabla f(x, y) = (\, f_x(x, y),\ f_y(x, y)\,) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

13. Know the derivative rules for the gradient as given in Theorem B on page 642.

14. The graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ has many different slopes, or "steepnesses." In fact, the graph will have a different slope in every different direction. The slope of the graph at the given point (x_0, y_0) and in a given direction $\mathbf{u}$ is called a **directional derivative**, and is denoted by $D_{\mathbf{u}} f(x_0, y_0)$. (See also item 4 above.)

15. Be able to explain why the following two equations are true.

$$D_{\mathbf{i}} f(x_0, y_0) = f_x(x_0, y_0)$$
$$D_{\mathbf{j}} f(x_0, y_0) = f_y(x_0, y_0)$$

16. We use the gradient vector to compute the directional derivative of $f(x, y)$ at a given point (x_0, y_0) in a given direction $\mathbf{u}$ (where $\mathbf{u}$ must be a unit vector, $||\mathbf{u}|| = 1$),

$$D_{\mathbf{u}} f(x_0, y_0) = \mathbf{u} \cdot \nabla f(x_0, y_0).$$

Notice how we are using unit vectors to describe direction.

17. Given a point (x_0, y_0), the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ has many different slopes, or "steepnesses." The maximal slope of the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ is in the direction of the gradient vector. Another way to put this is that the maximal directional derivative is in the direction of the gradient. Also, the maximal slope is equal to the length of the gradient,

$$D_{\mathbf{u}_{\nabla f}} f(x_0, y_0) = ||\nabla f(x_0, y_0)||,$$

where $\mathbf{u}_{\nabla f}$ is the unit vector in the direction of the gradient. The reason for these two facts is that for any direction $\mathbf{u}$,

$$D_{\mathbf{u}} f(x_0, y_0) = \mathbf{u} \cdot \nabla f(x_0, y_0) = ||\mathbf{u}|| \, ||\nabla f(x_0, y_0)|| \cos\theta = ||\nabla f(x_0, y_0)|| \cos\theta \le ||\nabla f(x_0, y_0)||.$$

So the most that $D_{\mathbf{u}} f(x_0, y_0)$ can be is $||\nabla f(x_0, y_0)||$, and $D_{\mathbf{u}} f(x_0, y_0) = ||\nabla f(x_0, y_0)||$ exactly when the angle between $\mathbf{u}$ and $\nabla f(x_0, y_0)$ is zero (that is, $\mathbf{u}$ points in the direction of $\nabla f(x_0, y_0)$).

18. Given a point (x_0, y_0), if we compute $c = f(x_0, y_0)$, then the gradient vector $\nabla f(x_0, y_0)$ is perpendicular to the level curve of height c. (Remember that level curves and the gradient vector are two-dimensional objects; they live in the two-dimensional xy-plane, not in the three-dimensional xyz-space.) Be sure to compare this fact with the previous item. The gradient points in the direction of steepest increase and the negative of the gradient points in the direction of steepest decrease. This item says that the direction of "no increase or decrease" is exactly "half way" between the steepest increase and the steepest decrease.

19. Let $\mathbf{x} = (x, y)$ denote an unknown point and let $\mathbf{x_0} = (x_0, y_0)$ denote a given point in the xy-plane. Then we can use the gradient vector to rewrite the formula for the plane tangent to the graph of $f(x, y) = f(\mathbf{x})$ at the point $(x_0, y_0, f(x_0, y_0)) = (\mathbf{x_0}, f(\mathbf{x_0}))$,

$$T(\mathbf{x}) = f(\mathbf{x_0}) + \nabla f(\mathbf{x_0}) \cdot (\mathbf{x} - \mathbf{x_0}).$$

We can also use a directional derivative to rewrite the tangent plane formula as

$$T(\mathbf{x}) = f(\mathbf{x_0}) + D_{\mathbf{u}} f(\mathbf{x_0}) \, ||\mathbf{x} - \mathbf{x_0}||$$

(where $\mathbf{u}$ is the unit vector in the direction of $\mathbf{x} - \mathbf{x_0}$). Notice how much each of these looks like the formula for the tangent *line* at the point $(x_0, f(x_0))$ for a function of *one* variable $f(x)$,

$$T(x) = f(x_0) + f'(x_0)(x - x_0).$$

20. We have two ways of describing surfaces in three-dimensional xyz-space. We can describe a surface as the graph of a function of two variables, $z = f(x, y)$ (as in Section 12.1). Or we can describe a surface as a level set of a function of three variables, $c = F(x, y, z)$, where c is a given constant (as in Sections 11.8 and 12.7).

21. Given a function of two variables $f(x, y)$, if we define a new function of three variables, $F(x, y, z)$, by $F(x, y, z) = z - f(x, y)$, then the zero level set $0 = F(x, y, z)$ is exactly the same surface as the graph of $z = f(x, y)$. This shows that graphs of functions of two variables are a special case of level sets for functions of three variables. (This fact is used in Section 12.7 of the textbook.)

22. You need to know what the normal vector at a point (x_0, y_0, z_0) on a surface is in each case (that is, the case of a graph $z = f(x, y)$ or the case of a level set $c = F(x, y, z)$). For case of a graph, the normal vector is given in item 8 above,

$$\mathbf{n} = (\,-f_x(x_0, y_0),\; -f_y(x_0, y_0),\; 1\,).$$

For the case of a level set, since a gradient vector is always perpendicular to its level set, the normal vector is the gradient vector of the function F,

$$\mathbf{n} = \nabla F(x_0, y_0, z_0) = (\,F_x(x_0, y_0, z_0),\; F_y(x_0, y_0, z_0),\; F_z(x_0, y_0, z_0)\,).$$

23. Know the chain rule for functions of several variables (Section 12.6). Know how to illustrate and derive a multi-variable chain rule using a picture of all the variable dependencies in a give problem.

24. Know the definitions, given at the top of page 660, of local and global maximums and minimums for a function of two variables $f(x, y)$.

25. Know the definition, given at the bottom of page 660, of a critical point for a function of two variables $f(x, y)$.

26. Know the Second Derivative Test, given on page 662, that determines if a critical point for a function $f(x, y)$ is a local maximum, a local minimum, a saddle point, or inconclusive. Compare this with the second derivative test for a function of one variable, $f(x)$, as given on page 202.