1. Be able to use the table of quadratic surfaces (pages 607-608) and "completing the square" to name and sketch a given equation of the form
$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

2. For a function of two variables $f(x, y)$, a **level curve** (or **level set**) of height c is the set of (x, y) points that solve the equation $c = f(x, y)$. Another way to define a level curve of height c is to intersect the graph of $z = f(x, y)$ with the horizontal plane $z = c$ and then project the intersection curve down to the xy-plane.

3. For a function of two variables $f(x, y)$, a **contour plot** is a simultaneous graph in the xy-plane of many level curves for f. You should be able to tell from a contour plot where the function f has a local maximum or local minimum, or where it has a "saddle point." You should also be able to tell from a contour plot where the function's graph is more or less steep.

4. For a function of two variables $f(x, y)$, a **vertical slice** is the intersection of the graph of $z = f(x, y)$ with a plane perpendicular to the xy-plane. Know how to use the idea of a vertical slice to describe what we mean by the "partial derivatives" and the "directional derivatives" of $f(x, y)$ at a given point (x_0, y_0).

5. For a function of two variables $f(x, y)$, know how to compute the partial derivatives $f_x(x, y)$ and $f_y(x, y)$, and, for a given point (x_0, y_0), know the meaning of $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$.

6. For a function of two variables $f(x, y)$, know how to compute the "higher order" and "mixed" partial derivatives.

7. For a function of two variables $f(x, y)$ and a point (x_0, y_0), know why the two vectors
$$(\,1,\,0,\,\frac{\partial f}{\partial x}(x_0, y_0)\,) \quad \text{and} \quad (\,0,\,1,\,\frac{\partial f}{\partial y}(x_0, y_0)\,)$$
are both tangent vectors to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

8. The cross product of the two vectors from the previous item,
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & f_x(x_0, y_0) \\ 0 & 1 & f_y(x_0, y_0) \end{vmatrix} = (\,-\frac{\partial f}{\partial x}(x_0, y_0),\,-\frac{\partial f}{\partial y}(x_0, y_0),\,1\,)$$
is perpendicular to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$, so it is a normal vector for the plane tangent to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

9. Let $\mathbf{n} = (\,-f_x(x_0, y_0),\,-f_y(x_0, y_0),\,1\,)$, let $\mathbf{x} = (x, y, z)$ denote an unknown point, and let $\mathbf{x_0} = (x_0, y_0, z_0)$ denote a given point. Then the vector equation $\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0$ for the plane tangent to the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ simplifies to
$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

10. The tangent plane formula is itself a function of x and y, $T_{(x_0,y_0)}(x,y)$, that we call the **tangent plane approximation function** for $f(x,y)$ at the point (x_0,y_0),

$$T_{(x_0,y_0)}(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0).$$

For arbitrary points (x,y) "near" the given point (x_0,y_0)

$$f(x,y) \approx T_{(x_0,y_0)}(x,y).$$

The tangent plane function approximates the original function.

11. Here is a picture illustrating the tangent plane approximation.

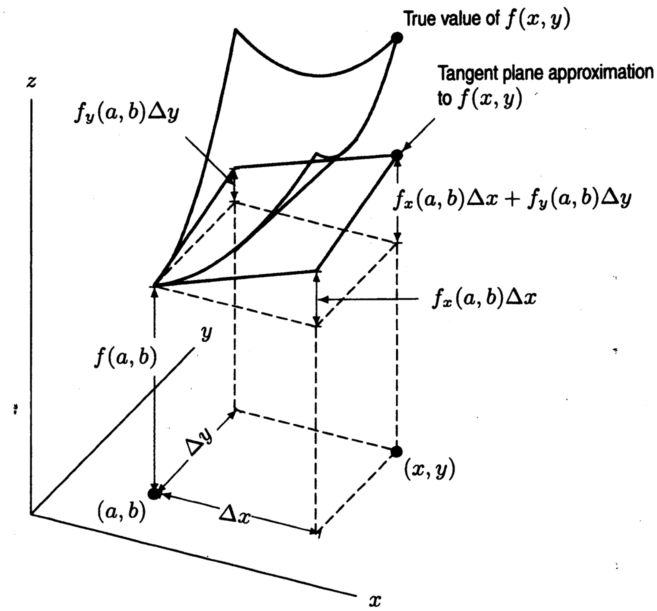

Figure 14.22: Local linearization: Approximating $f(x,y)$ by the z-value from the tangent plane

12. For a function of two variables $f(x,y)$, the **gradient vector** is defined to be the two-dimensional vector

$$\nabla f(x,y) = (\, f_x(x,y),\ f_y(x,y)\,) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

13. Know the derivative rules for the gradient as given in Theorem B on page 642.

14. We use the gradient vector to compute the directional derivative of $f(x,y)$ at a given point (x_0,y_0) in a given direction $\mathbf{u}$ (where $\mathbf{u}$ must be a unit vector, $||\mathbf{u}|| = 1$),

$$D_{\mathbf{u}}f(x_0,y_0) = \mathbf{u} \cdot \nabla f(x_0,y_0).$$

15. Given a point (x_0,y_0), if we compute $c = f(x_0,y_0)$, then the gradient vector $\nabla f(x_0,y_0)$ is perpendicular to the level curve of height c. (Remember that level curves and the gradient vector are two-dimensional objects; they live in the two-dimensional xy-plane, not in the three-dimensional xyz-space.)

16. Given a point (x_0, y_0), the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ has many different slopes, or "steepnesses." The maximal slope of the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ is in the direction of the gradient vector. Another way to put this is that the maximal directional derivative is in the direction of the gradient. Also, the maximal slope is equal to the length of the gradient,

$$D_{\mathbf{u}_{\nabla f}} f(x_0, y_0) = ||\nabla f(x_0, y_0)||$$

where $\mathbf{u}_{\nabla f}$ is the unit vector in the direction of the gradient.

17. Let $\mathbf{x} = (x, y)$ denote an unknown point and let $\mathbf{x_0} = (x_0, y_0)$ denote a given point in the xy-plane. Then we can use the gradient vector to rewrite the formula for the plane tangent to the graph of $f(x, y) = f(\mathbf{x})$ at the point $(x_0, y_0, f(x_0, y_0)) = (\mathbf{x_0}, f(\mathbf{x_0}))$,

$$T(\mathbf{x}) = f(\mathbf{x_0}) + \nabla f(\mathbf{x_0}) \cdot (\mathbf{x} - \mathbf{x_0}).$$

Notice how much this looks like the formula for the tangent *line* at the point $(x_0, f(x_0))$ for a function of *one* variable $f(x)$,

$$T(x) = f(x_0) + f'(x_0)(x - x_0).$$

18. We have two ways of describing surfaces in three-dimensional xyz-space. We can describe a surface as the graph of a function of two variables, $z = f(x, y)$. Or we can describe a surface as a level set of a function of three variables, $c = F(x, y, z)$, where c is a given constant. You need to know what the normal vector to a point (x_0, y_0, z_0) on the surface is in each case. For the first case, the normal vector is given in item 7 above,

$$\mathbf{n} = (\,-f_x(x_0, y_0),\ -f_y(x_0, y_0),\ 1\,).$$

For the second case, since a gradient vector is always perpendicular to its level set, the normal vector is the gradient vector of the function F,

$$\mathbf{n} = \nabla F(x_0, y_0, z_0) = (\,F_x(x_0, y_0, z_0),\ F_y(x_0, y_0, z_0),\ F_z(x_0, y_0, z_0)\,).$$

19. Know the chain rule for functions of several variables. Know how to illustrate and derive a multi-variable chain rule using a picture of all the variable dependencies in a give problem.