

CS 206 Exam 3 Review Problems

Fall, 2011

The exam will be over material from the following sections.

- Worksheet 5, Sections 5.5.1, 5.5.2, 5.6 and 5.7.

Problem 1: Part (a) A student wanted to graph a circle and used the following command. What result did the student get? What did the student do wrong? What would be the correct command?

```
[ > plot( [sin, cos], 0..2*Pi );
```

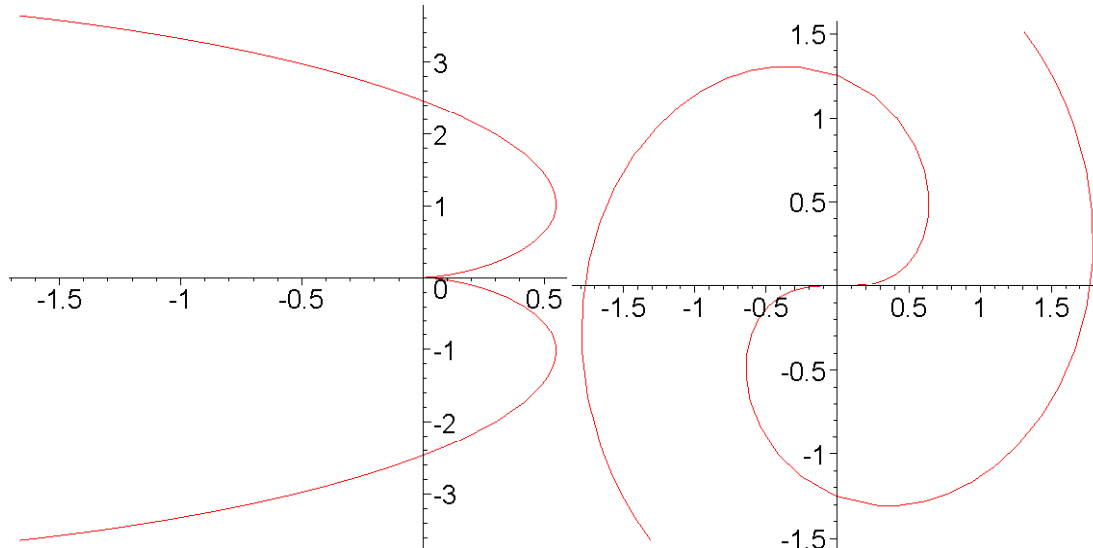
Part (b) A student wanted to graph a circle and used the following command. What result did the student get? What did the student do wrong? What would be the correct command?

```
[ > plot( x^2+y^2=1, x=-2..2, y=-2..2 );
```

Part (c) A student wanted to graph a circle and used the following command. What result did the student get? What did the student do wrong? What would be the correct command?

```
[ > plot( 1, theta=0..2*Pi );
```

Problem 2: The following two graphs are both "polar parabolas". Which one is a graph of $r = \theta^2$ in polar coordinates and which one is a graph of $\theta = r^2$ in polar coordinates? Give an explanation of how you can match these two formulas with the two graphs.



Problem 3: Draw a sketch of the graph that the following Maple command would produce.

```
[ > plot( [ [2,0], [2,Pi/2], [-1,-Pi/2], [-1,-Pi],  
>          [-1,Pi/2], [2,3*Pi/2], [2,2*Pi] ], coords=polar );
```

Problem 4: Part (a): What does the following command draw? Carefully explain why. Your explanation should include information about what it means to graph an equation, what the solution of $1 = r^2$ is, and how points in polar coordinates are plotted.

```
[ > plots[implicitplot]( 1=r^2, r=-2..5, theta=0..2*Pi, coords=polar  
);
```

Part (b): What does the following command draw? Carefully explain why. Your explanation should include information about what it means to graph an equation, what the solution of $1 = \theta^2$ is, and how points in polar coordinates are plotted.

```
[ > plots[implicitplot]( 1=theta^2, r=-2..5, theta=-Pi/2..Pi/2,  
  coords=polar );
```

Part (c): What does the following command draw? Carefully explain why. Your explanation should include information about what it means to graph an equation, what the solution of $0 = (r + 1)(r - 2)(r - 3)$ is, and how points in polar coordinates are plotted.

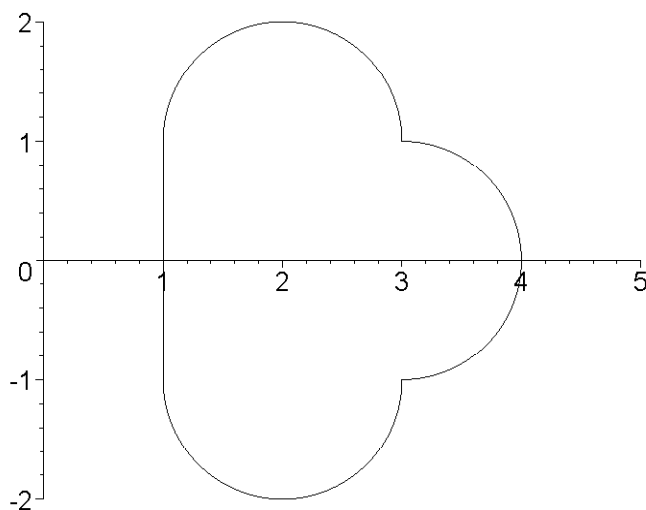
```
[ > plots[implicitplot]( 0=(r+1)*(r-2)*(r-3), r=-2..5, theta=0..2*Pi,  
  coords=polar );
```

Problem 5: Describe what the following animation would look like. Carefully explain why. In particular, explain what all of the brackets in the plot-valued function do.

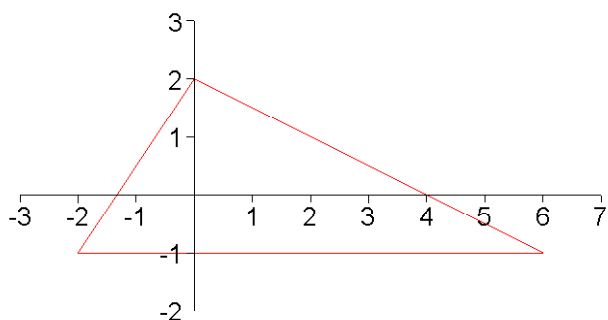
```
[ > p := t -> plot( [ [0,0], [1+cos(t),t] ], coords=polar ):  
[ > frames := seq( p(n*Pi/30), n=0..60 ):  
[ > plots[display]( frames, insequence=true );
```

Problem 6: Consider the following region in the xy -plane. The three curves in the boundary of the region are half circles.

- **Part (a):** Is this a "Type I" region? That is, can you define functions $f_1(x)$ and $f_2(x)$ that describe the "top" and "bottom" edges of the region? If not, explain why. If so, give formulas for the two functions.
- **Part (b):** Is this a "Type II" region? That is, can you define functions $g_1(y)$ and $g_2(y)$ that describe the "left" and "right" edges of the region? If not, explain why. If so, give formulas for the two functions.



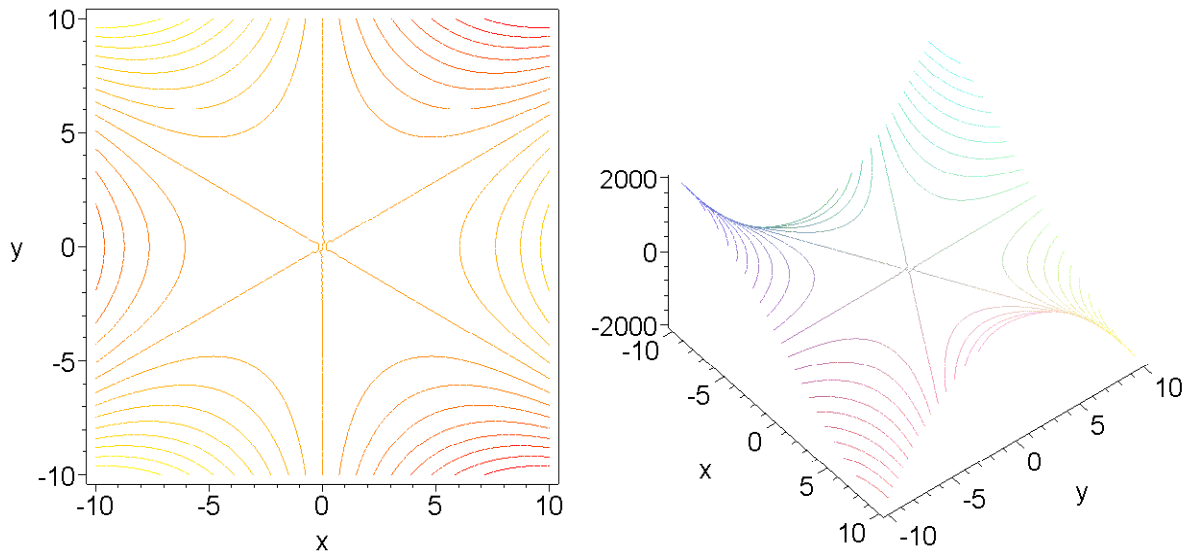
Problem 7: Below is a region in the plane in the shape of a triangle.



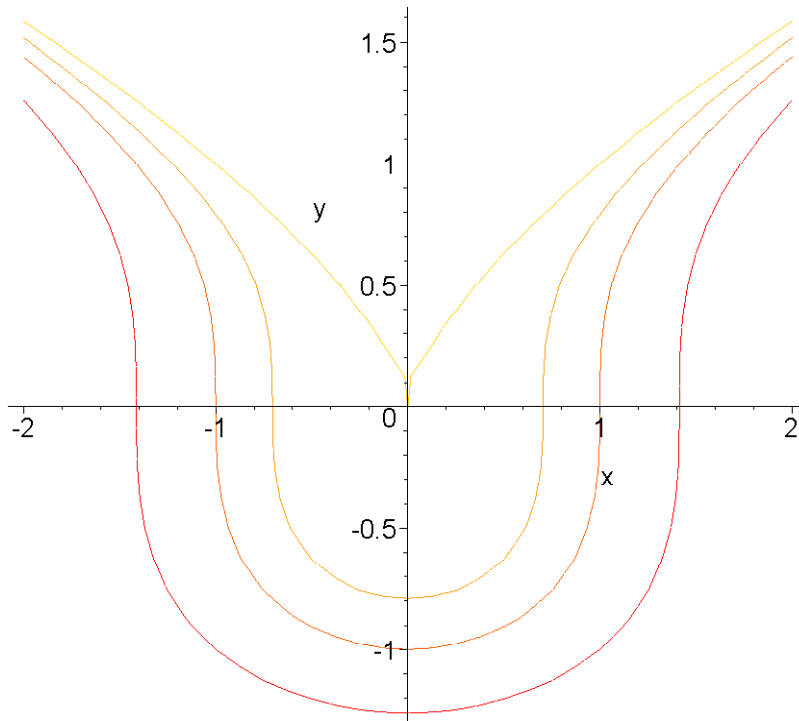
Part (a): Give the Maple command that would graph the function $f(x, y)$ over this region with the region described as a "Type I" region, with "Type I" defined as in the previous problem (do not worry about what the definition of $f(x, y)$ is).

Part (b): Give the Maple command that would graph the function $f(x, y)$ over this region with the region described as a "Type II" region, with "Type II" defined as in the previous problem (do not worry about what the definition of $f(x, y)$ is).

Problem 8: Below are two graphs of the function $f(x, y) = x^3 - 3xy^2$. In what ways are the two graphs alike and in what ways do they differ? What Maple command would be used to draw each graph?



Problem 9: Write out the full Maple command to draw a graph of the function $z = y^3 - x^2$ over the non rectangular region bounded by the level set $z = -1$. Here is a contour diagram for $y^3 - x^2$ showing contours for z equal to $-2, -1, -0.5$, and 0 .



Problem 10: Explain the difference between the following two Maple commands.

- `plot3d({x+2*y, 2*x-y, x*y}, x=-10..10, y=-10..10);`
- `plot3d([x+2*y, 2*x-y, x*y], x=-10..10, y=-10..10);`

Problem 11: Below are two different parameterizations of the sphere. Explain in detail how each one works.

```
[ > plot3d([sin(v)*cos(u), cos(v), sin(v)*sin(u)],
           u=0..2*Pi, v=0..Pi);
  > plot3d([cos(v)*sin(u), cos(v)*cos(u), sin(v)],
           u=0..Pi, v=0..2*Pi);
```

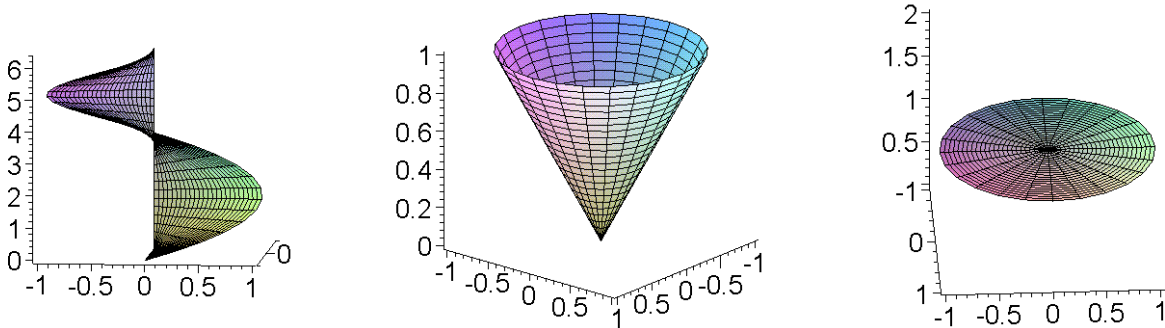
Problem 12: Suppose that the functions f_1 and g_1 parameterize a square in the plane and the functions f_2 and g_2 parameterize a triangle. If we start with the following parameterization of a sphere

$(u, v) \rightarrow [\sin(2\pi v) \cos(2\pi u), \sin(2\pi v) \sin(2\pi u), \cos(2\pi v)]$ with $u \in [0, 1]$ and $v \in [0, 1]$, then explain what surface is parameterized by each of the following two parameterizations.

- **(a):** $(u, v) \rightarrow [g_1(v) \cos(2\pi u), g_1(v) \sin(2\pi u), f_1(v)]$ with $u \in [0, 1]$ and $v \in [0, 1]$
- **(b):** $(u, v) \rightarrow [g_2(v)f_1(u), g_2(v)g_1(u), f_2(v)]$ with $u \in [0, 1]$ and $v \in [0, 1]$

Problem 13: First, match each of the following parameterizations with its surface. Second, for each parameterization, explain in detail how each component of the parameterization contributes to the shape of the surface being parameterized.

- `plot3d([u*cos(v), u*sin(v), 1], u=0..1, v=0..2*Pi)`
- `plot3d([u*cos(v), u*sin(v), u], u=0..1, v=0..2*Pi)`
- `plot3d([u*cos(v), u*sin(v), v], u=0..1, v=0..2*Pi)`



Problem 14: Let f be a real valued Maple function of two variables. The following `plot3d` command

- `plot3d(f(x,y), x=-2..2, y=-5..5)`

draws the exact same graph as one of the following three `plot3d` commands. Which one is it? Explain why. Also, explain how the graphs drawn by the other two `plot3d` commands differ from the graph drawn by the one above.

- `plot3d([x, f(x,y), y], x=-2..2, y=-5..5)`
- `plot3d([x, y, f(x,y)], x=-2..2, y=-5..5)`
- `plot3d([y, x, f(x,y)], x=-2..2, y=-5..5)`