CS 206 Exam 1 Review Problems

Fall 2011

This exam will be over material from the following sections.

- Worksheet 1, Sections 1.2, 1.3.1, 1.3.2, 1.3.3, 1.5.
- Worksheet 2, Sections 2.2, 2.3.
- Worksheet 3, Section 3.2.
- Worksheet 4, Sections 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8.

Problem 1: What would be the results of the following two commands?

```
> subs( x=xy, xy+x/y );
> factor( % );
```

Problem 2: The following command makes use of an equal sign and the result has another equal sign in it.

```
> solve( a=(1+b)/x^2, {b} );
```

$$\{ b = a\, x^2 - 1 \}$$

Do each of these two equal signs have the same meaning?

Problem 3: The following command makes use of two equal signs and the result has a third equal sign in it.

```
> subs( b=-1+a*x^2, a=(1+b)/x^2 );
```

$$a = a$$

What can you say about each of these three uses of an equal sign. Do each of these equal signs have the same meaning (i.e., do they represent an assignment or an equation)?

Problem 4: Do the following two formulas have the same meaning? Explain your answer.

$$y = x^2 - 1$$
$$x^2 - y = 1$$

Problem 5: Explain three differences between the Maple commands `solve` and `fsolve`.

Problem 6: The following function definition is not correct and will produce an error message from Maple. Explain why Maple cannot allow a function definition like this one. Based on this example, what rule do you think is true about the definitions of Maple functions?

```
> f := (x,x) -> x^2+x^3;
```

Problem 7: One student claims that the following two functions are the same (since they use the same formula) but another student claims that they are different. Who is right and explain why.

```
> (u,v) -> 3*u+v^2;
> (v,u) -> 3*u+v^2;
```

Problem 8: The following four commands define two different functions. Which commands define the same function? Explain why.

```
> (x,y) -> cos(2*x)+y^2;
> (y,x) -> cos(2*x)+y^2;
> (y,x) -> cos(2*y)+x^2;
> (u,v) -> cos(2*v)+u^2;
```

Problem 9: In standard mathematical notation we use juxtaposition to denote multiplication, as in $2\,x$ to mean 2 multiplied with x, or $y\,(x+1)$ to mean y times the quantity $x+1$, or $(u+v)\,(x-y)$ to mean the quantity $u+v$ times the quantity $x-y$. Maple notation does not allow this simple and common symbolism for multiplication. Explain how Maple interprets the following two commands.

```
> y(x+1);
> (u+v)(x-y);
```

Problem 10: Carefully explain how Maple arrives at the following result.

```
> solve( {solve(x+y-z=0, z), 3*x-2*y=1}, {x,y} );
```

$$\{y = \frac{-1}{5}, x = \frac{1}{5}\}$$

Explain why the following command produces an error message (the only change from the last command was to put a pair of braces around the `z`).

```
> solve( { solve(x+y-z=0, {z}), 3*x-2*y=1 }, {x,y} );
Error, (in solve) invalid arguments
```

Problem 11: Explain what is wrong with the following `plot` command and then fix it by changing only one character in the command. (There are two ways to solve this problem.)

```
> plot( (z->z^2+1)(y), z=-3..3 );
Plotting error, empty plot
```

Problem 12: If you get the following error message, what should you suspect is the problem?

```
> plot( z^2+1, z=-3..3 );
Error, (in plot) invalid arguments
```

Problem 13: What is wrong with the combination of the following two commands? How can this be fixed?

```
> solve( t^3-t=1, t );
> assign( % );
```

Problem 14: Suppose that we make the following assignment in Maple.

```
> f := t -> piecewise( t<=-2, t^2, t<0, t^3, t<=1, t-1, t>1, 4 );
```

- (a) What would be the value of $f(0)$? Explain why.
- (b) What would be the value of the function f at 1/2? Explain why.
- (c) What would be the value of $f(-3)$? Explain why.

Problem 15: Explain why the following two piecewise defined functions are really the same function.

```
> f := x -> piecewise( x<0, x, x<=5, 2*x, x>5, -x+15 );
> g := x -> piecewise( x>5, -x+15, x>=0, 2*x, x<0, x );
```

Problem 16: Suppose we are given the following piecewise defined function

```
> f := x -> piecewise(x<-1, 2*x, x<=1, x^2, x<=3, 1/x, x>3, ln(x));
> f(x);
```

$$\begin{cases} 2x & x < -1 \\ x^2 & x \le 1 \\ \dfrac{1}{x} & x \le 3 \\ \ln(x) & 3 < x \end{cases}$$

Fill in the missing conditions in the following definition of **g** so that **g** is equivalent to **f**.

```
> g := x -> piecewise( ???, ln(x), ???, 1/x, ???, x^2, ???, 2*x );
```

Problem 17: Write out the definition, using correct Maple syntax, for a piecewise defined Maple function of one variable whose graph is a line with slope -1 and *y*-intercept 1 for negative values of the independent variable, one period of the cosine function for inputs between 0 and 2 π, and a horizontal line of height 3 for inputs greater than or equal to 2 π.

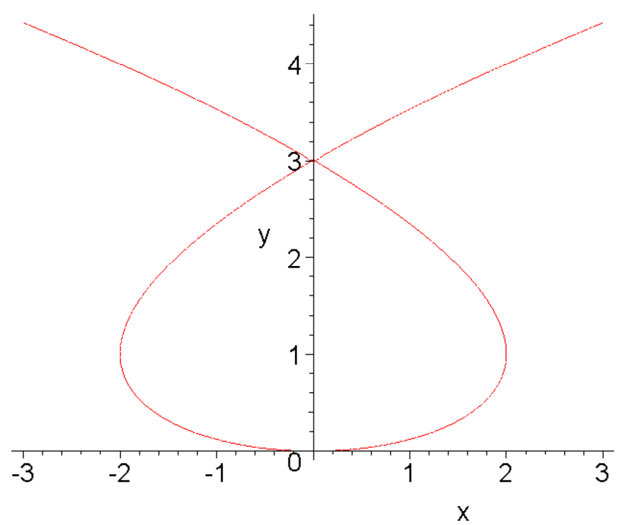
Problem 18: Consider the following equation.

$$x^2 = y^3 - 6\,y^2 + 9\,y$$

Below is what the graph of this equation looks like. Find two different ways to decompose this graph into the graphs of three implicitly defined functions with *y* as a function of *x*. Find two different ways to decompose this graph into two graphs of implicitly defined functions with *x* as a function of *y*. Draw sketches of your implicitly defined functions. (All together you will draw 10 sketches for 10 implicitly defined functions.) For each implicitly defined function, state what its domain and codomain are (write the domain and codomain down near the sketch of each function).

```
> x^2=y^3-6*y^2+9*y;
> plots[implicitplot]( %, x=-3..3, y=-1..5, grid=[100,100] );
```

$$x^2 = y^3 - 6\,y^2 + 9\,y$$



Problem 19: Use the classification of function from Section 4.2.2 to classify each of the following functions.

- (a) $(x \rightarrow \ln(x)) + (y \rightarrow \sin(y))$
- (b) $f(x, y) = (\ln(x) + \sin(y), \ln(x) + \sin(y), \ln(x) + \sin(y))$
- (c) $\ln + \sin$
- (d) $x \rightarrow (\ln(x), \sin(x))$
- (e) $\ln(x) + \sin(y)$
- (f) $(x, y) \rightarrow (\ln(x), \sin(y))$
- (g) $u \rightarrow (\ln(u), \sin(u), 0)$
- (h) $x \rightarrow \ln(y) + \sin(x)$

Exercise 20: How many *different* Maple functions of three variables can you define using the expression `w*exp(u+v)` on the right hand side of the arrow operator? How could you use Maple to demonstrate that your functions really are different from each other?

Exercise 21: How many *different* Maple functions of three variables can you define using the expression `w+u^2+v^3` on the right hand side of the arrow operator? How could you use Maple to demonstrate that your functions really are different from each other?